HW 4.4 Key

1. Let T be the lifetime in years of an electronic device. Assume that T follows a uniform distribution on the interval $[2,12]$. Given that the device is 4.1 years old, find the probability that it will fail within the next 2.4 years.

$\boxed{\text{A)}}$ 0.3038 B) 0.3220 C) 0.3403 D) 0.3585 E) 0.3767

$$P[T<6.5 \mid T>4.1] = \frac{6.5-4.1}{12-4.1} = \boxed{0.3038}$$

2. Assume that X is a random variable that follows a continuous uniform distribution. Given that $P[X>9]=\frac{5}{8}$ and $P[X>15]=\frac{1}{4}$, find $Var[X]$.

$\boxed{\text{A)}}$ 21.33 B) 20.05 C) 22.61 D) 23.89 E) 25.17

$X \sim \text{UNIF}(a,b)$

$$P[X>9] = \frac{b-9}{b-a} = \frac{5}{8}$$

$$P[X>15] = \frac{b-15}{b-a} = \frac{1}{4}$$

$$\frac{b-9}{b-15} = \frac{5}{2} \Rightarrow b = 19$$

$$a = 3$$

$$Var[X] = \frac{16^2}{12} = \boxed{21.33}$$

3. Assume that X is a random variable that follows a continuous uniform distribution on the interval $[3,b]$. Given that $E[X]=8Var[X]$, find b.

$\boxed{\text{A)}}$ 5.53 B) 4.87 C) 5.20 D) 5.86 E) 6.19

$$E[X] = 8\,Var[X] \Rightarrow \frac{b+3}{2} = 8\frac{(b-3)^2}{12}$$

$$\Rightarrow 6(b+3) = 8(b-3)^2$$

$$\Rightarrow 6b+18 = 8b^2-48b+72$$

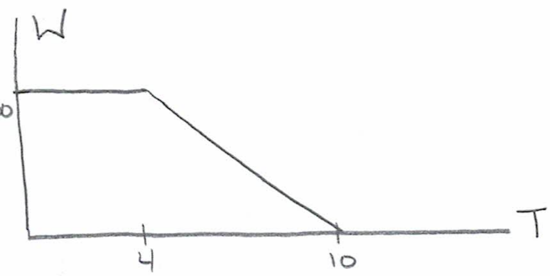
$$\Rightarrow 8b^2-54b+54 = 0$$

$$\Rightarrow b = 1.2207 \quad \text{or} \quad \boxed{b = 5.5292}$$

4. Let T be the time until failure for a newly purchased appliance. Assume that T is uniformly distributed on the interval $[0,16]$. A warranty on the appliance will make a payment to the owner if it fails within 10 years of being purchased. The amount paid by the warranty is a constant \$600 during the first 4 years, and then decreases linearly until 10 years after purchase, at which point it pays 0. Find the expected value of the warranty payment, rounded to the nearest dollar.

262.5

A) ~~152~~ B) 134 C) 139 D) 143 E) 148



$$W = \begin{cases} 600 & \text{if } T < 4 \\ 1000 - 100T & \text{if } 4 < T < 10 \\ 0 & \text{if } T > 10 \end{cases}$$

$$E[W] = \int_0^4 600\left(\tfrac{1}{16}\right)dt + \int_4^{10}(1000-100t)\left(\tfrac{1}{16}\right)dt$$

$$= 600\left(\tfrac{1}{16}\right)(4) + \tfrac{1}{16}\left[1000t - 50t^2\right]_4^{10}$$

$$= 150 + \tfrac{1}{16}\left[5000 - 3200\right]$$

$$= \boxed{262.5}$$

5. A machine in a factory requires frequent shutdowns for maintenance. Let T denote the amount of time, in days, between shut-downs. Assume that T is uniformly distributed on the interval $[20,60]$. The cost of performing the maintenance increases as the amount of time between shutdowns increases. In particular, if C is the cost of maintenance during a shutdown, then $C = 200e^{0.04t}$. Find the expected value of the maintenance costs during a shutdown.

A) 1100 B) 1067 C) 1133 D) 1166 E) 1199

$$E[C] = \int_{20}^{60} 200e^{0.04t}\left(\tfrac{1}{40}\right)dt$$

$$= \int_{20}^{60} 5e^{0.04t}\,dt$$

$$= \left[125e^{0.04t}\right]_{20}^{60}$$

$$= \boxed{1099.70}$$