HW 4.5(a) Key

1. Let X be a normally distributed variable with mean 41 and standard deviation 7. Find $P[31.9 < X < 45.06]$.

$\boxed{\text{A)}}$ 0.6222 B) 0.5476 C) 0.5725 D) 0.5974 E) 0.6471

$$P[31.9 < X < 45.06] = P\left[\frac{31.9-41}{7} < Z < \frac{45.06-41}{7}\right]$$

$$= P[-1.3 < Z < 0.58] = \Phi(0.58) - \Phi(-1.3)$$

$$= 0.7190 - 0.0968 = \boxed{0.6222}$$

2. Let X be a normally distributed variable with mean 46 and standard deviation 8. Given that $X > 40.8$, find the prob. that $\cancel{P}[X > 56.24]$.

$\boxed{\text{A)}}$ 0.1351 B) 0.1148 C) 0.1216 D) 0.1284 E) 0.1419

$$P[X > 56.24 \mid X > 40.8] = \frac{P[X > 56.24]}{P[X > 40.8]} = \frac{P[Z > 1.28]}{P[Z > -0.65]}$$

$$= \frac{\Phi(-1.28)}{\Phi(0.65)} = \frac{0.1003}{0.7422} = \boxed{0.1351}$$

3. Let X be a normally distributed variable with mean 1000 and standard deviation 100. Given that $P[X < 850] = 0.0668$ and $P[X < 1112] = 0.8686$, find $P[X < 1036]$.

$\boxed{\text{A)}}$ 0.6406 B) 0.6726 C) 0.7046 D) 0.7367 E) 0.7687

$$P[X < 850] = 0.0668 \Rightarrow \frac{850-\mu}{\sigma} = -1.5$$

$$P[X < 1112] = 0.8686 \Rightarrow \frac{1112-\mu}{\sigma} = 1.12$$

$$\begin{aligned} 850 - \mu &= -1.5\sigma \\ 1112 - \mu &= 1.12\sigma \end{aligned} \Rightarrow 262 = 2.62\sigma \Rightarrow \begin{aligned} \sigma &= 100 \\ \mu &= 1000 \end{aligned}$$

$$P[X < 1036] = P[Z < 0.36] = \boxed{0.6406}$$

4. Suppose that annual salaries for full-time workers in a certain city are normally distributed with mean $43,000. You are given that 91.95% of full-time workers in the city earn at least $35,500 per year. Find the 78th percentile of annual salaries in the city.

A) $47,150 B) $48,550 C) $50,000 D) $51,400 E) $52,800

$S \sim N(\mu = 43000, \sigma^2 = ?)$

$P[S > 35500] = 0.9195 \Rightarrow P[S < 35500] = 0.0805$

$\Rightarrow P\left[Z < \frac{35500 - 43000}{\sigma}\right] = 0.0805 \Rightarrow \frac{-7500}{\sigma} = -1.40$

$\sigma = 5357.14$

$P[Z < z] = 0.78 \Rightarrow z = 0.772$

$\frac{\pi_{0.78} - 43000}{5357.14} = 0.772 \Rightarrow \pi_{0.78} = \boxed{\$47,135}$

5. Assume that X and Y are normally distributed random variables with the same mean, μ. The variance of X is 2.807 times the variance of Y. The 77.94th percentile of X is equal to the pth percentile of Y. Find p.

A) 90.15 B) 84.74 C) 86.54 D) 88.34 E) 91.95

$X \sim N(\text{mean} = \mu, \text{sd} = \sqrt{2.807}\,\sigma)$

$Y \sim N(\text{mean} = \mu, \text{sd} = \sigma)$

Let $k = \pi^{X}_{0.7794} = \pi^{Y}_{p}$

$P[X < k] = 0.7794$ and $P[Y < k] = p$

$P[X < k] = 0.7794 \Rightarrow \frac{k - \mu}{\sqrt{2.807}\,\sigma} = 0.77 \Rightarrow \frac{k-\mu}{\sigma} = 1.29$

$P[Y < k] = P\left[Z < \frac{k-\mu}{\sigma}\right] = P[Z < 1.29]$

$= \boxed{0.9015}$