HW 4.5(b) Key

1. Let X be a normally distributed variable with a mean of 1350 and a standard deviation of 250. Two independent observations of X are drawn. Find the probability that the smaller of the two observations is less than 1185.

$\boxed{\text{A)}}$ 0.4444 B) 0.3555 C) 0.3778 D) 0.4000 E) 0.4222

$$P[\text{Smaller obv.} < 1185] = P[\text{At least one obv.} < 1185] = 1 - P[\text{Both obv.} > 1185]$$
$$= 1 - (P[X > 1185])^2 = 1 - (P[Z > -0.66])^2$$
$$= 1 - (P[Z < 0.66])^2 = 1 - (0.7454)^2$$
$$= \boxed{0.4444}$$

2. A factory manufactures ball bearings. The diameter of the bearings produced are normally distributed with a mean of 305 mm and a standard deviation of 3.3 mm. A bearing must be discarded if its diameter is greater than 311.336 mm, or less than 298.664 mm. A batch of 21 ball bearings is produced. Find the probability that fewer than three bearings in the batch had to be discarded.

$\boxed{\text{A)}}$ 0.8949 B) 0.7087 C) 0.7553 D) 0.8018 E) 0.8483

D = Bearing Diameter $D \sim N(\mu = 305, \sigma = 3.3)$

$$p = P[X < 298.664] + P[X > 311.336] = P[Z < -1.92] + P[Z > 1.92] = 0.0548$$

N = Number of discarded bearings $N \sim BIN(n = 21, p = 0.0548)$

$$P[N \le 2] = q^{21} + 21pq^{20} + 210p^2q^{19} = \boxed{0.8951}$$

3. Assume that the heights of the eruptions in a particular geyser are normally distributed with a mean of 72 ft, and that the heights of different eruptions are independent of one another. The probability that two successive eruptions are both greater than 47 ft is equal to 0.7606. Find the 68th percentile of the eruption heights.

$\boxed{\text{A)}}$ 82.34 B) 86.62 C) 90.90 D) 95.19 E) 99.47

X = eruption height $X \sim N(\mu = 72, \sigma = ?)$

$$P[2 \text{ eruptions} > 47] = (P[X > 47])^2 = 0.7606$$

$$P[X > 47] = 0.8721 \Rightarrow P[X < 47] = 0.1279$$

$$\frac{47 - 72}{\sigma} = -1.136 \Rightarrow \sigma = 22$$

$$P[X < \pi_{0.68}] = 0.68 \Rightarrow \frac{\pi_{0.68} - 72}{22} = 0.468 \Rightarrow \pi_{0.68} = \boxed{82.296}$$

4. The number of times that a geyser erupts in ~~an hour~~ a day follows a Poisson distribution with a mean of 159. The height of the geyser's eruptions are normally distributed with a mean of 58 ft, and a standard deviation of 19 ft. Assume that the heights of different eruptions are independent of one another. Find the probability that there are no eruptions with a height greater than 109 ft in a given day.

$\boxed{\text{A)}}$ 0.5610 B) 0.4735 C) 0.5027 D) 0.5319 E) 0.5902

N = # of eruptions , $N \sim POI(\lambda = 159)$

X = Height of eruption, $X \sim N(\mu = 58, \sigma = 19)$

$$p = P[X < 109] = P[Z < 2.6842] = 0.9964$$

$$P[\text{None w/ } X > 109] = \sum_{n=0}^{\infty} P[n \text{ eruptions, and all } n \text{ have } X < 109]$$

$$= \sum_{n=0}^{\infty} \left[\frac{e^{-159}\, 159^n}{n!}\right] p^n = \sum_{n=0}^{\infty}\left[\frac{e^{-159}(159p)^n}{n!}\right]$$

There is a slight discrepency here due to the way p was rounded.

$$= \frac{e^{-159}}{e^{-159p}} \sum_{n=0}^{\infty} \frac{e^{-159p}(159p)^n}{n!} = e^{-159+159p} = \boxed{0.5642}$$

Note: $\frac{e^{-159p}(159p)^n}{n!}$ is the pmf for $POI(\lambda = 159p)$. Thus, the sum in the last line is equal to 1.

5. Let X be a normally distributed random variable with mean 2200 and standard deviation σ. There is a constant k such that $P[X < k] = 0.9573$ and $P[X < k | X > 1870] = 0.9529$. Find k.

$\boxed{\text{A)}}$ 2630 B) 2462 C) 2546 D) 2714 E) 2798

$$P[X < k] = 0.9573 \Rightarrow \frac{k - 2200}{\sigma} = 1.72$$

$$P[X < k | X > 1870] = \frac{P[1870 < X < k]}{P[X > 1870]} = \frac{P[X<k] - P[X < 1870]}{1 - P[X < 1870]} = 0.9529$$

$$\Rightarrow 0.9573 - P[X < 1870] = 0.9529 - 0.9529\, P[X < 1870]$$

$$\Rightarrow 0.0471\, P[X < 1870] = 0.0044$$

$$\Rightarrow P[X < 1870] = 0.0934$$

$$\Rightarrow \frac{1870 - 2200}{\sigma} = -1.32 \Rightarrow \sigma = 250$$

$$\Rightarrow \frac{k - 2200}{250} = 1.72 \Rightarrow k = \boxed{2630}$$