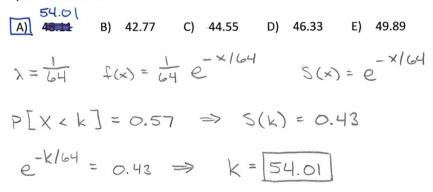
HW 4.6(b) Key

1. Let *X* be a continuous random variable that follows an exponential distribution with a mean of 77. Calculate P[X > 9 | X < 34].

[A] 0.6910 B) 0.6143 C) 0.6398 D) 0.6654 E) 0.7165
$$\lambda = \frac{1}{77} \qquad f(x) = \frac{1}{77} e^{-x/77} \qquad S(x) = e^{-x/77} \qquad F(x) = 1 - e^{-x/77}$$

$$P[x_{79} | x_{79} | x_{79}$$

2. Let X be a continuous random variable that follows an exponential distribution with a mean of M. Find the 57th percentile of X.



3. A mechanical device contains two electrical motors. The life spans of the two motors each follows an exponential distribution with a mean of 22 months, and are independent of one another. The device is able to function with a single motor. Given that the machine fails within 24 months, find the probability that it fails within 11 months.

A) 0.3511 B) 0.3661 C) 0.3812 D) 0.3963 E) 0.4114

$$T_1 = I_1 f_{espan} \text{ of motor } I$$
 $T_2 = I_1 f_{espan} \text{ of motor } Z$
 $T_{13} T_{2} \sim E \times P(\lambda = \frac{1}{22})$
 $X = I_{14} f_{espan} \text{ of device}$
 $F(t) = I - e^{-t/22}$
 $P[X < II] | X < Z < I] = \frac{P[X < II]}{P[X < 24]} = \frac{P[T_{14} < II] \text{ and } T_{24} < II]}{P[T_{14} < II]}$
 $= \frac{[F(II)]^2}{[F(Z < II)]^2} = [O.35II]$

4. A mechanical device contains two electrical motors. The life spans of the two motors each follows an exponential distribution with a mean of 18 months, and are independent of one another. The device is able to function with a single motor. Given that at least one of the engines fail within the first 24 months, find the probability that the device fails within 38 months.

0.8083 A) 0.3301

B) 0.7587

C) 0.7944

D) 0.8658

E) 0.9015

$$T_1 = 1_1 fespan \text{ of motor } 1$$
, $T_2 = 1_1 fespan \text{ of motor } 2$, $T_1, T_2 \sim EXP(\lambda = \frac{1}{8})$
 $X = 1_1 fespan \text{ of device}$ $F(t) = 1 - e^{-t \cdot 1/8}$

$$= \frac{P[T_1 < 38 \text{ and } T_2 < 24] + P[T_1 < 24 \text{ and } T_2 < 38] - P[T_1 < 24 \text{ and } T_2 < 24]}{P[T_1 < 24] + P[T_2 < 24] - P[T_1 < 24 \text{ and } T_2 < 24]}$$

$$= \frac{2F(24)F(38) - [F(24)]^2}{2F(24) - F(24)^2} = \frac{2F(38) - F(24)}{2 - F(24)}$$

5. Two geysers that are near to each other are labelled as Geyser A and Geyser B. For each geyser, the time between its eruptions follows a exponential distribution. The eruption times of the two geysers are independent of one another. The mean amount of time between eruptions of Geyser A is 36 minutes, and the mean amount of time between eruptions of Geyser B is 62 minutes. Given that exactly one of the geysers erupts within the next 52 minutes, determine the probability that it will be Geyser A.

A) 0.7115

B) 0.7421 C) 0.7727

$$T_1 = t_1 me$$
 until Geyser A erupts, $T_2 = t_1 me$ until Geyser B erupts
 $T_1 \sim EXP(\chi = \frac{1}{36})$ $T_2 \sim EXP(\chi = \frac{1}{62})$
 $F_1(t) = 1 - e^{-t/36}$ $F_2(t) = 1 - e^{-t/62}$

$$= \frac{F_1(52) S_2(52)}{F_2(52) S_2(52)} = [0.7115]$$