HW 4.6(b) Key

1. Let *X* be a continuous random variable that follows an exponential distribution with a mean of 77. Calculate $P[X > 9 | X < 34]$.

$\boxed{\text{A)}}$ 0.6910 B) 0.6143 C) 0.6398 D) 0.6654 E) 0.7165

$\lambda = \frac{1}{77}$ $f(x) = \frac{1}{77} e^{-x/77}$ $S(x) = e^{-x/77}$ $F(x) = 1 - e^{-x/77}$

$$P[x > 9 | x < 34] = \frac{P[9 < x < 34]}{P[x < 34]} = \frac{F(34) - F(9)}{F(34)}$$

$$= \boxed{0.6910}$$

2. Let *X* be a continuous random variable that follows an exponential distribution with a mean of 64. Find the 57th percentile of X.

$\boxed{\text{A)}}$ ~~48.11~~ 54.01 B) 42.77 C) 44.55 D) 46.33 E) 49.89

$\lambda = \frac{1}{64}$ $f(x) = \frac{1}{64} e^{-x/64}$ $S(x) = e^{-x/64}$

$P[X < k] = 0.57 \Rightarrow S(k) = 0.43$

$e^{-k/64} = 0.43 \Rightarrow k = \boxed{54.01}$

3. A mechanical device contains two electrical motors. The life spans of the two motors each follows an exponential distribution with a mean of 22 months, and are independent of one another. The device is able to function with a single motor. Given that the machine fails within 24 months, find the probability that it fails within 11 months.

$\boxed{\text{A)}}$ 0.3511 B) 0.3661 C) 0.3812 D) 0.3963 E) 0.4114

T_1 = lifespan of motor 1 T_2 = lifespan of motor 2 $T_1, T_2 \sim EXP(\lambda = \frac{1}{22})$

X = lifespan of device $F(t) = 1 - e^{-t/22}$

$$P[x < 11 | x < 24] = \frac{P[x < 11]}{P[x < 24]} = \frac{P[T_1 < 11 \text{ and } T_2 < 11]}{P[T_1 < 24 \text{ and } T_2 < 24]}$$

$$= \frac{[F(11)]^2}{[F(24)]^2} = \boxed{0.3511}$$

4. A mechanical device contains two electrical motors. The life spans of the two motors each follows an exponential distribution with a mean of 18 months, and are independent of one another. The device is able to function with a single motor. Given that at least one of the ~~engines~~ motors fail within the first 24 months, find the probability that the device fails within 38 months.

0.8083

$\boxed{\text{A)}}$ ~~[illegible]~~ B) 0.7587 C) 0.7944 D) 0.8658 E) 0.9015

T_1 = lifespan of motor 1, T_2 = lifespan of motor 2, $T_1, T_2 \sim EXP\left(\lambda = \frac{1}{18}\right)$

X = lifespan of device $F(t) = 1 - e^{-t/18}$

$$P[X < 38 \mid T_1 < 24 \text{ or } T_2 < 24] = \frac{P[X < 38 \text{ and } (T_1 < 24 \text{ or } T_2 < 24)]}{P[T_1 < 24 \text{ or } T_2 < 24]}$$

$$= \frac{P[T_1 < 38 \text{ and } T_2 < 24] + P[T_1 < 24 \text{ and } T_2 < 38] - P[T_1 < 24 \text{ and } T_2 < 24]}{P[T_1 < 24] + P[T_2 < 24] - P[T_1 < 24 \text{ and } T_2 < 24]}$$

$$= \frac{2F(24)F(38) - [F(24)]^2}{2F(24) - F(24)^2} = \frac{2F(38) - F(24)}{2 - F(24)}$$

$$= \boxed{0.8083}$$

5. Two geysers that are near to each other are labelled as Geyser A and Geyser B. For each geyser, the time between its eruptions follows a exponential distribution. The eruption times of the two geysers are independent of one another. The mean amount of time between eruptions of Geyser A is 36 minutes, and the mean amount of time between eruptions of Geyser B is 62 minutes. Given that exactly one of the geysers erupts within the next 52 minutes, determine the probability that it will be Geyser A.

$\boxed{\text{A)}}$ 0.7115 B) 0.7421 C) 0.7727 D) 0.8033 E) 0.8339

T_1 = time until Geyser A erupts, T_2 = time until Geyser B erupts

$T_1 \sim EXP\left(\lambda = \frac{1}{36}\right)$ $T_2 \sim EXP\left(\lambda = \frac{1}{62}\right)$

$F_1(t) = 1 - e^{-t/36}$ $F_2(t) = 1 - e^{-t/62}$

$$P[T_1 < 52 \mid (T_1 < 52 \text{ and } T_2 > 52) \text{ or } (T_1 > 52 \text{ and } T_2 < 52)]$$

$$= \frac{P[T_1 < 52 \text{ and } T_2 > 52]}{P[T_1 < 52 \text{ and } T_2 > 52] + P[T_1 > 52 \text{ and } T_2 < 52]}$$

$$= \frac{F_1(52)S_2(52)}{F_1(52)S_2(52) + S_1(52)F_2(52)} = \boxed{0.7115}$$