

HW 5.2(a) Key

1. Let X and Y be discrete random variables with joint probability mass function given by

$$f(x, y) = \frac{1}{293}(6-x)(9+y) \text{ for } x=1, 2, 3 \text{ and } x \leq y \leq 3. \text{ Find } P[X < Y].$$

- A) 0.5563 B) 0.5062 C) 0.5229 D) 0.5396 E) 0.5730

$$\begin{aligned} P[X < Y] &= f(1, 2) + f(1, 3) + f(2, 3) \\ &= \frac{1}{293} [5(11) + 5(12) + 4(12)] \\ &= \boxed{0.5563} \end{aligned}$$

2. Let X and Y be discrete random variables with joint probability mass function given by

$$f(x, y) = \frac{1}{165}(10-x)(1+y) \text{ for } x=1, 2, 3 \text{ and } x \leq y \leq 3. \text{ Find } \text{Var}[X+Y].$$

- A) 1.450 B) 1.493 C) 1.537 D) 1.580 E) 1.624

X	1	1	1	2	2	3
Y	1	2	3	2	3	3
$f(x, y)$	$18/165$	$27/165$	$36/165$	$24/165$	$32/165$	$28/165$
$X+Y$	2	3	4	5	6	
p	$18/165$	$27/165$	$60/165$	$32/165$	$28/165$	

using
"1-var stats"

$$\begin{aligned} \text{Var}[X+Y] &= (1.20406)^2 \\ &= \boxed{1.450} \end{aligned}$$

3. Let X and Y be discrete random variables with joint probability mass function given by

$$f(x, y) = (0.68^x)(0.32^y) \text{ for } x=1, 2, 3, \dots \text{ and } y=1, 2, 3, \dots \text{ Find } P[X+Y \geq 4].$$

- A) 0.5648 B) 0.4970 C) 0.5309 D) 0.5987 E) 0.6326

$$\begin{aligned} P[X+Y \geq 4] &= 1 - P[X+Y \leq 3] \\ &= 1 - f(1, 1) - f(1, 2) - f(2, 1) \\ &= 1 - 0.68(0.32) - 0.68(0.32)^2 - (0.68)^2(0.32) \\ &= \boxed{0.5648} \end{aligned}$$

4. Let X and Y be discrete random variables with joint probability mass function given by $f(x,y) = (r^x)(6.6^y)$ for $x = 0, 1, 2, 3, \dots$ and $y = 0, 1, 2, 3, \dots$. Find r .

A) 1.1786 B) 1.0371 C) 1.1079 D) 1.2493 E) 1.3200

$$\sum_{x=0}^{\infty} \sum_{y=0}^{\infty} r^x (6.6)^y = \sum_{x=0}^{\infty} r^x \sum_{y=0}^{\infty} 6.6^y$$

$$= \frac{1}{1-r} \frac{1}{1-6.6} = 1$$

$$\Rightarrow 1 = (1-r)(-5.6) \Rightarrow 1 = -5.6 + 5.6r$$

$$\Rightarrow 5.6r = 6.6 \Rightarrow r = \boxed{1.1786}$$

This problem is flawed. This is not a valid pmf.

The "correct" answer is

$$\frac{b}{b-1} \text{ where}$$

b is the base in the 2nd exponential.

(i.e. $b=6.6$ here)

5. Let X and Y be discrete random variables with joint probability mass function given by $f(x,y) = k(4.2 + xy)$ for $x = 2, 3, 4$ and $1 \leq y \leq x$, where y is an integer. Find $P[X - Y = 1]$.

A) 0.5415 B) 0.5090 C) 0.5253 D) 0.5578 E) 0.5740

$$f(2,1) + f(3,1) + f(3,2) + f(4,1) + f(4,2) + f(4,3) = 1$$

$$k [(4.2 + 2) + (4.2 + 3) + (4.2 + 6) + (4.2 + 4) + (4.2 + 8) + (4.2 + 12)] = 1$$

$$k [6(4.2) + (2 + 3 + 6 + 4 + 8 + 12)] = 1$$

$$k = \frac{1}{60.2}$$

$$P[X - Y = 1] = f(2,1) + f(3,2) + f(4,3)$$

$$= \frac{1}{60.2} [(4.2 + 2) + (4.2 + 6) + (4.2 + 12)]$$

$$= \boxed{0.5415}$$