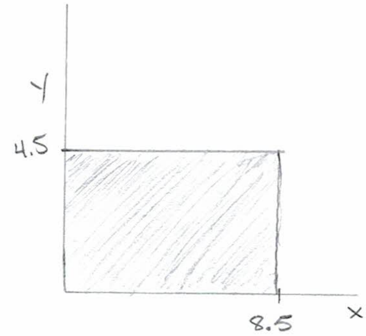
HW 5.2(b) Key



1. Let X and Y be continuous random variables with joint probability density function given by $f(x,y)=k(x+y^2)$ for $0\le x\le 8.5$ and $0\le y\le 4.5$. Find $P[X>4, Y>3.2]$.

$\boxed{\text{A)}}$ 0.2949 B) 0.3038 C) 0.3126 D) 0.3215 E) 0.3303

$$k\int_0^{8.5}\int_0^{4.5}(x+y^2)\,dy\,dx = k\int_0^{8.5}\left[xy+\tfrac{1}{3}y^3\right]_0^{4.5}dx$$

$$= k\int_0^{8.5}\left[4.5x+30.375\right]dx = k\left[2.25x^2+30.375x\right]_0^{8.5}$$

$$= k(420.75)=1 \Rightarrow k = 1/420.75$$

$$P[X>4, Y>3.2] = k\int_4^{8.5}\int_{3.2}^{4.5}(x+y^2)\,dy\,dx = k\int_4^{8.5}\left[xy+\tfrac{1}{3}y^3\right]_{3.2}^{4.5}dx$$

$$= k\int_4^{8.5}\left[1.3x+19.452333\right]dx = k\left[0.65x^2+19.452333x\right]_4^{8.5}$$

$$= k\left[0.65(56.25)+19.452333(4.5)\right] = \boxed{0.2949}$$

2. Let X and Y be continuous random variables with joint probability density function given by $f(x,y)=\frac{1}{56}e^{-(x/7+y/8)}$ for $x,y\ge 0$. Find $E\left[X^2Y\right]$.

$\boxed{\text{A)}}$ 784 B) 760 C) 808 D) 831 E) 855

$$E[X^2Y] = \int_0^\infty\int_0^\infty x^2y\left[\tfrac{1}{56}e^{-x/7-y/8}\right]dy\,dx$$

$$= \int_0^\infty \tfrac{1}{7}x^2e^{-x/7}dx\int_0^\infty \tfrac{1}{8}ye^{-y/8}dy$$

$$= \left[e^{-x/7}(x^2+14x+98)\right]_\infty^0\left[e^{-y/8}(y+8)\right]_\infty^0$$

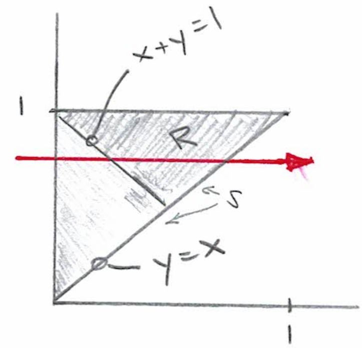
$$= (98-0)(8-0)$$

$$= \boxed{784}$$

		$\frac{1}{7}e^{-x/7}$
+	x^2	$-e^{-x/7}$
−	$2x$	$7e^{-x/7}$
+	2	$-49e^{-x/7}$
−	0	*

		$\frac{1}{8}e^{-y/8}$
+	y	$-e^{-y/8}$
−	1	$8e^{-y/8}$
+	0	*

3. Let X and Y be continuous random variables with joint probability density function given by $f(x,y)=\dfrac{1}{3.09}(2.18+6y)$ for $0<x<y<1$. Find $P[X+Y>1]$.



A) 0.5809 B) 0.5461 C) 0.5635 D) 0.5983 E) 0.6158

$$P[X+Y>1] = \int_{0.5}^{1}\int_{1-y}^{y} \frac{1}{3.09}(2.18+6y)\,dx\,dy$$

$$= \frac{1}{3.09}\int_{0.5}^{1}\left[2.18x+6yx\right]_{1-y}^{y} dy$$

$$= \frac{1}{3.09}\int_{0.5}^{1}\left[(2.18y+6y^2)-(2.18-2.18y+6y-6y^2)\right] dy$$

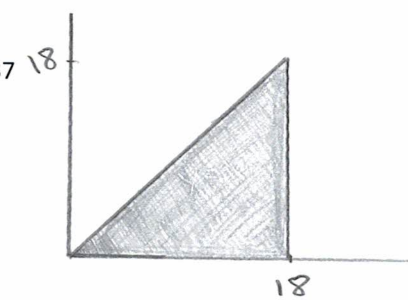
$$= \frac{1}{3.09}\int_{0.5}^{1}\left[12y^2-1.64y-2.18\right] dy$$

$$= \frac{1}{3.09}\left[4y^3-0.82y^2-2.18y\right]_{0.5}^{1}$$

$$= \frac{1}{3.09}\left[1-(-0.795)\right] = \boxed{0.5809}$$

4. Let X and Y be continuous random variables with joint probability density function given by $f(x,y)=k$ for $0\le y\le x\le 18$. Find $E\left[e^{x/4}\right]$.

A) ~~31.216~~ 31.2158 B) 28.094 C) 29.655 D) 32.777 E) 34.337



$$k(\text{Area}) = 1 \Rightarrow k = \frac{1}{0.5(18)^2} = \frac{1}{162}$$

$$E\left[e^{x/4}\right] = \int_0^{18}\int_0^{x} e^{x/4}\left(\frac{1}{162}\right) dy\,dx$$

$$= \frac{1}{162}\int_0^{18}\left[y e^{x/4}\right]_0^{x} dx$$

$$= \frac{1}{162}\int_0^{18} x e^{x/4}\,dx$$

		$e^{x/4}$
+	x	$4e^{x/4}$
−	1	$16e^{x/4}$
+	0	*

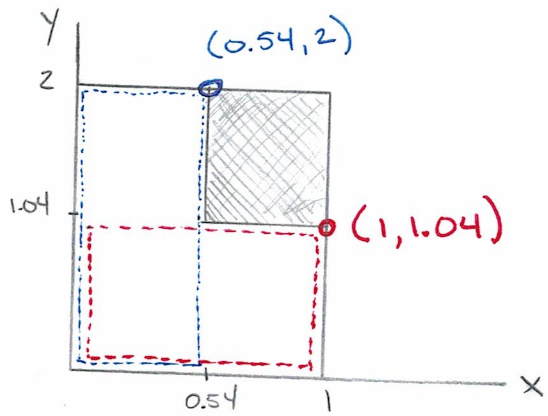
$$= \frac{1}{162}\left[e^{x/4}(4x-16)\right]_0^{18}$$

$$= \frac{1}{162}\left[e^{4.5}(56)+16\right]$$

$$= \boxed{31.2158}$$

5. Let X and Y be continuous random variables with joint cumulative distribution function given by $F(x,y)=\dfrac{x^3y^2}{20}+\dfrac{xy^3}{10}$ for $0\le x\le 1$ and $0\le y\le 2$. Find $P[X>0.54, Y>1.04]$.

A) 0.4392 B) 0.3953 C) 0.4172 D) 0.4612 E) 0.4831



$P[X>0.54, Y>1.04] = 1 - F(0.54, 2) - F(1, 1.04) + F(0.54, 1.04)$

$= 1 - 0.4635 - 0.1666 + 0.0693$

$= \boxed{0.4392}$