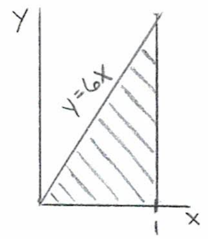
HW 5.3(b) Key

1. Let *X* and *Y* be continuous random variables with joint probability density function given by $f(x,y) = \frac{9}{2}xy$ for $0 \le x \le 1$ and $0 \le y \le 6x$. Find the conditional pdf of X given that $Y = 5.2$.



$$f_Y(y) = \int_{y/6}^{1} \frac{9}{2} xy \, dx = \left[\frac{9}{4} x^2 y \right]_{y/6}^{1}$$

$$= \frac{9}{4}\left[y - \frac{y^3}{36} \right]$$

$$f_Y(5.2) = \frac{9}{4}\left[5.2 - (5.2)^3/36 \right] = 2.912$$

$$g(x \mid Y=5.2) = \frac{f(x,5.2)}{f_Y(5.2)} = \frac{23.4x}{2.912}$$

$$g(x \mid Y=5.2) = \boxed{8.0357x}$$

2. Let *X* and *Y* be continuous random variables with joint probability density function given by $f(x,y) = 7.2e^{-(x+7.2y/x)}$ for $x, y \ge 0$. Find the conditional pdf of Y given that $X = 12$.

$$f_X(x) = \int_0^\infty 7.2e^{-x-7.2y/x} \, dy = \left[7.2^2 \left(\tfrac{1}{x}\right) e^{-x-7.2y/x} \right]_\infty^0$$

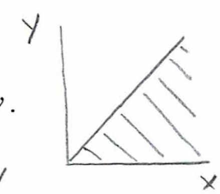
$$= (7.2)^2 \left[\tfrac{1}{x} e^{-x} - 0 \right] = (7.2)^2 \tfrac{1}{x} e^{-x}$$

$$f_X(12) = (7.2)^2 \left(\tfrac{1}{12}\right) e^{-12}$$

$$h(y \mid x=12) = \frac{f(x,12)}{f_X(12)} = \frac{7.2\, e^{-12} e^{-7.2y/12}}{(7.2)^2\left(\frac{1}{12}\right)e^{-12}} = \frac{7.2}{12} e^{-7.2y/12}$$

$$= \boxed{0.6\, e^{-0.6y}}$$

3. Let *X* and *Y* be continuous random variables with joint probability density function given by $f(x,y) = 104e^{-(8x+5y)}$ for $0 < y < x < \infty$. Find the conditional pdf of X given that $Y = y$.

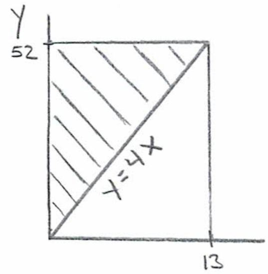


$$f_Y(y) = \int_y^\infty 104 e^{-8x} e^{-5y} \, dx = \left[13e^{-8x} e^{-5y} \right]_\infty^y = 13e^{-13y}$$

$$g(x \mid Y=y) = \frac{f(x,y)}{f_Y(y)} = \frac{104 e^{-8x} e^{-5y}}{13 e^{-13y}}$$

$$= \boxed{8 e^{-8x+8y}}$$

4. Let X and Y be continuous random variables that are uniformly distributed on the region defined by $0 \le x \le 13$ and $4x \le y \le 52$. Find the conditional pdf of Y given that $X = x$.



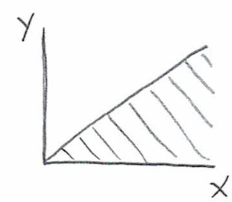
$$\text{Area} = 13(52)/2 = 338 \qquad f(x,y) = 1/338$$

$$f_X(x) = \int_{4x}^{52} \frac{1}{338}\,dy = \frac{1}{338}(52-4x)$$

$$h(y|X=x) = \frac{f(x,y)}{f_X(x)} = \frac{\frac{1}{338}}{\frac{1}{338}(52-4x)} = \boxed{\frac{1}{52-4x}}$$

5. The continuous random variable X follows a gamma distribution with $\alpha = 3$ and $\beta = 3.6$. Given that $X = x$, the random variable Y is uniformly distributed on the interval $[0, x]$. Find the marginal probability density function for Y.

$$f_X(x) = \frac{3.6^3}{2!} x^2 e^{-3.6x} = 0.5(3.6)^3 x^2 e^{-3.6x}$$



$$h(y|X=x) = \frac{1}{x}$$

$$f(x,y) = h(y|x=x) f_x(x) = 0.5(3.6)^3 x e^{-3.6x}$$

$$f_Y(y) = \int_y^{\infty} 0.5(3.6)^3 x e^{-3.6x}\,dx$$

$$= \left[e^{-3.6x}\left(0.5(3.6)^2 x + 0.5(3.6)\right)\right]_{\infty}^{y}$$

$$= e^{-3.6y}(6.48y + 1.8)$$

$$= \boxed{1.8 e^{-3.6y}(3.6y+1)}$$

		$0.5(3.6)^3 e^{-3.6X}$
+	X	$-0.5(3.6)^2 e^{-3.6X}$
−	1	$0.5(3.6) e^{-3.6X}$
+	0	*