HW 5.7 Key

1. Let X and Y be discrete random variables with joint probability mass function given by the table below. Find $\text{Cor}[X,Y]$.

		X			
		1	2	4	
Y	2	0.22	0.05	0.04	0.31
	4	0.02	0.09	0.58	0.69
		0.24	0.14	0.62	

$\boxed{\text{A)}}$ 0.7419 B) 0.7196 C) 0.7642 D) 0.7864 E) 0.8087

Using 1-var stats: $E[X] = 3$ $\sigma_X = 1.3115$

$E[Y] = 3.38$ $\sigma_Y = 0.9250$

$$E[XY] = 2(0.22) + 4(0.05 + 0.02) + 8(0.04 + 0.09) + 16(0.58)$$
$$= 11.04$$

$$\text{Cov}[X,Y] = 11.04 - 3(3.38) = 0.9$$

$$\text{Cor}[X,Y] = \frac{0.9}{(1.3115)(0.9250)} = \boxed{0.7419}$$

2. Let X and Y be continuous random variables with joint probability density function given by $f(x,y) = \frac{1}{105}(x+y)$ on the region given by $0 \le x \le 3,\ 0 \le y \le 7$. Find $\text{Cov}[X,Y]$.

$\boxed{\text{A)}}$ −0.1225 B) −0.1164 C) −0.1286 D) −0.1347 E) −0.1409

$$E[X] = \frac{1}{105}\int_0^3\int_0^7 (x^2 + xy)\,dy\,dx = \frac{1}{105}\int_0^3 \left[x^2y + \tfrac{1}{2}xy^2\right]_0^7 dx$$
$$= \frac{1}{105}\int_0^3 \left[7x^2 + 24.5x\right] dx = \frac{1}{105}\left[\tfrac{7}{3}x^3 + 12.25x^2\right]_0^3 = 1.65$$

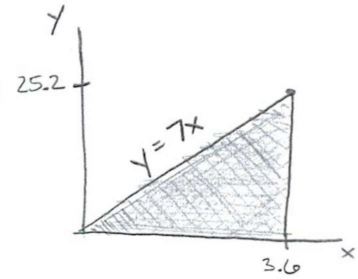
$$E[Y] = \frac{1}{105}\int_0^3\int_0^7 (xy + y^2)\,dy\,dx = \frac{1}{105}\int_0^3 \left[\tfrac{1}{2}xy^2 + \tfrac{1}{3}y^3\right]_0^7 dx$$
$$= \frac{1}{105}\int_0^3 \left[24.5x + \tfrac{343}{3}\right] dx = \frac{1}{105}\left[12.25x^2 + \tfrac{343}{3}x\right]_0^3 = 4.3167$$

$$E[XY] = \frac{1}{105}\int_0^3\int_0^7 (x^2y + xy^2)\,dy\,dx = \frac{1}{105}\int_0^3 \left[\tfrac{1}{2}x^2y^2 + \tfrac{1}{3}xy^3\right]_0^7 dx$$
$$= \frac{1}{105}\int_0^3 \left[24.5x^2 + \tfrac{343}{3}x\right] dx = \frac{1}{105}\left[\tfrac{24.5}{3}x^3 + \tfrac{343}{6}x^2\right]_0^3 = 7$$

$$\text{Cov}[X,Y] = 7 - 1.65(4.3167) = \boxed{-0.1225}$$

3. Let X and Y be continuous random variables that are uniformly distributed on the region given by $0 \le y \le 7x \le \cancel{3.6}$. Find $\mathrm{Cov}[X,Y]$.

25.2



$\boxed{A)}$ 2.5200   B) 2.3940   C) 2.6460   D) 2.7720   E) 2.8980

$$\text{Area} = \tfrac{1}{2}(3.6)(25.2) = 45.36 \qquad f(x,y) = \frac{1}{45.36}$$

$$E[X] = \frac{1}{45.36}\int_0^{3.6}\int_0^{7x} x\,dy\,dx = \frac{1}{45.36}\int_0^{3.6} 7x^2\,dx = 2.4$$

$$E[Y] = \frac{1}{45.36}\int_0^{3.6}\int_0^{7x} y\,dy\,dx = \frac{1}{45.36(2)}\int_0^{3.6} 49x^2\,dx = 8.4$$

$$E[XY] = \frac{1}{45.36}\int_0^{3.6}\int_0^{7x} xy\,dy\,dx = \frac{1}{45.36}\int_0^{3.6}\left[\tfrac{1}{2}xy^2\right]_0^{7x} dx$$

$$= \frac{1}{45.36}\int_0^{3.6}(24.5x^3)\,dx = \frac{1}{45.36}\left[6.125x^4\right]_0^{3.6} = 22.68$$

$$\mathrm{Cov}[X,Y] = 22.68 - 2.4(8.4) = \boxed{2.52}$$

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4. Let X and Y be exponentially distributed random variables with means of 6 and 24, respectively. Assume that $\rho_{X,Y} = 0.54$. Find $E[XY]$.

$\boxed{A)}$ ~~66.24~~ 221.76   B) 60.94   C) 63.59   D) 68.89   E) 71.54

$$E[X] = 6 \Rightarrow \lambda_X = \frac{1}{6} \Rightarrow \sigma_X = 6$$

$$E[Y] = 24 \Rightarrow \lambda_Y = \frac{1}{24} \Rightarrow \sigma_Y = 24$$

$$\mathrm{Cov}[X,Y] = \rho_{XY}\,\sigma_X\,\sigma_Y = 0.54(6)(24) = 77.76$$

$$E[XY] = \mathrm{Cov}[X,Y] + E[X]E[Y] = \boxed{221.76}$$

5.

Let X and Y be discrete random variables with probability mass function given by $f(x,y)=\dfrac{50x+y^2}{520}$, for all positive integers x and y satisfying $x+y\le 4$. Given that $E[X]=\dfrac{79}{40}$ and $E[Y]=\dfrac{199}{130}$, find $\mathrm{Cov}[X,Y]$.

A) –0.3175 B) –0.2599 C) –0.2723 D) –0.2846 E) –0.2970

Sample Space: (1,1) (1,2) (1,3) (2,1) (2,2) (3,1)

$$E[XY] = \frac{1}{520}\Big[1(50+1) + 2(50+4) + 3(50+9) + 2(100+1) + 4(100+4) + 3(150+1)\Big]$$

$$= \frac{1407}{520}$$

$$\mathrm{Cov}[X,Y] = \frac{1407}{520} - \frac{79}{40}\cdot\frac{199}{130} = \boxed{-0.3175}$$