HW 5.8 Key

1. Let X and Y be random variables such that $\text{Var}[X]=4$, $\text{Var}[Y]=7$, and $\text{Var}[X+Y]=19$. Let $Z=8X+9Y+3$. Find $\text{Var}[Z]$

A) 1399 B) 1343 C) 1371 D) 1427 E) 1455

$$\text{Var}[X+Y] = \text{Var}[X]+\text{Var}[Y]+2\,\text{Cov}[X,Y]$$

$$19 = 4+7+2\,\text{Cov}[X,Y] \Rightarrow \text{Cov}[X,Y]=4$$

$$\text{Var}[Z] = \text{Var}[8X+9Y] =$$

$$= 64\,\text{Var}[X]+81\,\text{Var}[Y]+2(8)(9)\,\text{Cov}[X,Y]$$

$$= \boxed{1399}$$

2. Let X and Y be random variables. You are given that:
   * $\text{Var}[X+8Y]=5129$
   * $\text{Var}[2X+4Y]=2084$
   * $\text{Var}[7X+6Y]=9721$

   Find $\text{Cov}[X,Y]$.

A) 44 B) 31 C) 35 D) 40 E) 48

$$\text{Var}X + 64\,\text{Var}Y + 16\,\text{Cov}[X,Y] = 5129$$

$$4\text{Var}X + 16\,\text{Var}Y + 16\,\text{Cov}[X,Y] = 2084$$

$$49\text{Var}X + 36\,\text{Var}Y + 84\,\text{Cov}[X,Y] = 9721$$

$$\left[\begin{array}{ccc|c} 1 & 64 & 16 & 5129 \\ 4 & 16 & 16 & 2084 \\ 49 & 36 & 84 & 9721 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 64 & 16 & 5129 \\ 0 & -240 & -48 & -18432 \\ 0 & -3100 & -700 & -241600 \end{array}\right] \rightarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 3.2 & 213.8 \\ 0 & 1 & 0.2 & 76.8 \\ 0 & 0 & -80 & -3520 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 73 \\ 0 & 1 & 0 & 68 \\ 0 & 0 & 1 & 44 \end{array}\right]$$

$$\text{Cov}[X,Y] = \boxed{44}$$

3. Let X and Y be random variables such that $\text{Var}[X]=4.5$, $\text{Var}[Y]=2.4$, and $\rho_{X,Y}=0.4$. Let $U=2X+8Y$ and $V=9X-5Y$. Find $\rho_{U,V}$.

A) 0.2600 B) 0.2548 C) 0.2652 D) 0.2704 E) 0.2756

$$\text{Cov}[X,Y] = \rho_{XY}\sigma_X\sigma_Y = 0.4\sqrt{4.5(2.4)} = 1.3145$$

$$\text{Var}[U] = 4\text{Var}[X] + 64\text{Var}[Y] + 32\,\text{Cov}[X,Y] = 213.6651$$

$$\text{Var}[V] = 81\text{Var}[X] + 25\text{Var}[Y] - 90\,\text{Cov}[X,Y] = 306.1919$$

$$\begin{aligned}\text{Cov}[U,V] &= \text{Cov}[2X+8Y,\ 9X-5Y] \\ &= 18\,\text{Cov}[X,X] - 10\,\text{Cov}[X,Y] + 72\,\text{Cov}[Y,X] - 40\,\text{Cov}[Y,Y] \\ &= 18\text{Var}[X] - 40\text{Var}[Y] + 62\,\text{Cov}[X,Y] = 66.5011\end{aligned}$$

$$\rho_{UV} = \frac{\text{Cov}[U,V]}{\sigma_U\sigma_V} = \frac{66.5011}{\sqrt{213.6651(306.1919)}} = \boxed{0.2600}$$

4. Let X and Y be random variables such that $\text{Var}[X]=6.5$, $\text{Var}[Y]=2.6$, and $\text{Cov}[X,Y]=2.7$. Let $U=2X+4Y$ and $V=kX+3Y$. The variables U and V are independent. Find k.

A) ~~-2.03~~ -1.99 B) −2.09 C) −2.15 D) −2.21 E) −2.27

$$\begin{aligned}\text{Cov}[U,V] &= \text{Cov}[2X+4Y,\ kX+3Y] \\ &= 2k\,\text{Var}[X] + 12\text{Var}[Y] + (6+4k)\,\text{Cov}[X,Y] = 0\end{aligned}$$

$$\Rightarrow 13k + 31.2 + 16.2 + 10.8k = 0$$

$$\Rightarrow \boxed{-1.99}$$

5. Let X, Y, and Z be random variables such that:

* $\text{Var}[X]=12$, $\text{Var}[Y]=11$, and $\text{Var}[Z]=7$
* $\text{Cov}[X,Y]=9$, $\text{Cov}[X,Z]=4$, and $\text{Cov}[Y,Z]=7$.

Find the variance of $W=8X+6Y+9Z+2$.

A) 3927 B) 3691 C) 3809 D) 4045 E) 4163

$$\begin{aligned}\text{Var}[W] &= \text{Var}[8X+6Y+9Z] \\ &= 64\text{Var}[X] + 36\text{Var}[Y] + 81\text{Var}[Z] + 96\,\text{Cov}[X,Y] + 144\,\text{Cov}[X,Z] + 108\,\text{Cov}[Y,Z] \\ &= \boxed{3927}\end{aligned}$$