Actex – Ch. 01

Addition Rule

$P(A \cup B)$

Actex – Ch. 01

Addition Rule

$P(A \cup B \cup C)$

Actex – Ch. 01

Mutually Exclusive Events

A and B are mutually exclusive if:

(Give two criteria)

Actex – Ch. 01

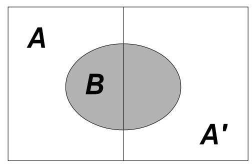
Complementary Events

$P(A')$

Actex – Ch. 01

Law of Total Probability

$P(B) = ?$



Actex – Ch. 01

DeMorgan's Laws

$P\left[(A \cup B)'\right]$

$P\left[(A \cap B)'\right]$

Actex – Ch. 02

Conditional Probability

$P(A \mid B)$

Actex – Ch. 02

Multiplication Rule

$P(A \cap B)$

Actex – Ch. 02

Independent Events

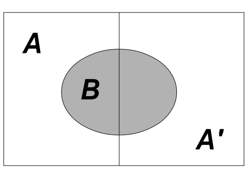
A and B are independent iff:

- $P(A \cap B) = ?$
- $P(A \mid B) = ?$
- $P(B \mid A) = ?$

Actex – Ch. 02

Baye's Rule

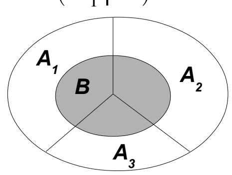
$P(A \mid B) = ?$



Actex – Ch. 02

Baye's Rule

$P(A_1 \mid B) = ?$



Actex – Ch. 02

Misc Probability Rules

$P(A' \mid B)$

Actex – Ch. 02

Misc Probability Rules

$P(A \cup B \mid C)$

$P(A \cap B) = 0$ $P(A \cup B) = P(A) + P(B)$	$P(A \cup B \cup C) =$ $P(A) + P(B) + P(C)$ $- P(A \cap B) - P(A \cap C)$ $- P(B \cap C) + P(A \cap B \cap C)$	$P(A \cup B) =$ $P(A) + P(B) - P(A \cap B)$
$P\left[(A \cup B)'\right] = P(A' \cap B')$ $P\left[(A \cap B)'\right] = P(A' \cup B')$	$P(B) = P(B \cap A) + P(B \cap A')$	$P(A') = 1 - P(A)$
$P(A \cap B) = P(A)P(B)$ $P(A \mid B) = P(A)$ $P(B \mid A) = P(B)$	$P(A \cap B) = P(A \mid B)P(B)$ $P(A \cap B) = P(B \mid A)P(A)$	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$
$P(A' \mid B) = 1 - P(A \mid B)$	$P(A_1 \mid B) =$ $\frac{P(B \mid A_1)P(A_1)}{P(B \mid A_1)P(A_1) + P(B \mid A_2)P(A_2) + P(B \mid A_3)P(A_3)}$	$P(A \mid B) =$ $\frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A')P(A')}$
		$P(A \cup B \mid C) =$ $P(A \mid C) + P(B \mid C) - P(A \cap B \mid C)$

Actex – Ch. 03	Actex – Ch. 03	Actex – Ch. 03
Permutations Find the number of ways of ordering n distinct objects.	Permutations Find the number of ways selecting an ordered collection of k objects from a pool of n distinct objects.	Permutations Find the number of ways of ordering n objects, which are of t different types.
Actex – Ch. 03		
Combinations Find the number of ways of selecting k objects from a pool of n distinct objects if order doesn't matter.		
Actex – Ch. 04	Actex – Ch. 04	Actex – Ch. 04
Probability Density Function X is a cont. random variable with pdf $f(x)$. Find $P[a \leq x \leq b]$.	Probability Density Function List the criteria for $f(x)$ to be a probability density function.	Cumulative Distribution Function Define CDF, $F(x)$, and the Survival Function, $S(x)$, in terms of the probabilities they represent.
Actex – Ch. 04	Actex – Ch. 04	Actex – Ch. 04
Cumulative Distribution Function Give $F(x)$ and $S(x)$ for a continuous random variable X with pdf $f(x)$.	Cumulative Distribution Function Find $F'(x)$ and $S'(x)$.	Cumulative Distribution Function Give $P[a < x \leq b]$ in terms of $F(x)$ and $S(x)$.
Actex – Ch. 04		
Hazard Rate Give two formulas for the hazard rate (or failure rate) $h(x)$.		

$\binom{n}{n_1 n_2 \cdots n_t} = \frac{n!}{n_1! \cdot n_2! \cdot \ldots \cdot n_t!}$	$nPk = \frac{n!}{(n-k)!}$ $= n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-k+1)$	$n!$
		$nCk = \binom{n}{k} = \frac{n!}{k! \cdot (n-k)!}$
$F(x) = P[X \le x]$ $S(x) = P[x > x]$ Note: $S(x) = 1 - F(x)$	1) $f(x) \ge 0$ for all x 2) $\int_{-\infty}^{\infty} f(x)\,dx = 1$	$P[a \le x \le b] = \int_a^b f(x)\,dx$
$P[a < x \le b] = F(b) - F(a)$ $P[a < x \le b] = S(a) - S(b)$	$F'(x) = f(x)$ $S'(x) = -f(x)$	$F(x) = \int_{-\infty}^{x} f(t)\,dt$ $S(x) = \int_x^{\infty} f(t)\,dt = 1 - F(x)$
		$h(x) = \frac{f(x)}{1 - F(x)}$ $h(x) = -\frac{d}{dx} \ln[1 - F(x)]$

Actex – Ch. 05	Actex – Ch. 05	Actex – Ch. 05
Expected Value Give $E[X]$ in terms of $f(x)$. (Discrete and continuous)	Expected Value Give $E[h(X)]$. (Discrete and Continuous)	Expected Value State the Darth Vader Rule.
Actex – Ch. 05	Actex – Ch. 05	Actex – Ch. 05
Moments 1) Define the *n*th moment of *X*. 2) Define the *n*th central moment of *X*.	Variance Give two formulas for $\mathrm{Var}[X]$.	Algebraic Properties $E[aX+b]=?$ $\mathrm{Var}[aX+b]=?$
Actex – Ch. 05	Actex – Ch. 05	Actex – Ch. 05
Moment Generating Functions Define $M_X(t)$.	Moment Generating Functions Give $E[X^n]$ in terms of $M_X(t)$.	Moment Generating Functions Give $\mathrm{Var}[X]$ in terms of $M_X(t)$.
Actex – Ch. 05	Actex – Ch. 05	Actex – Ch. 05
Moment Generating Functions Let $Y=aX+b$. Find $M_Y(t)$ in terms of $M_X(t)$.	Moment Generating Functions Assume *X* and *Y* are independent and let $S=X+Y$. Find $M_S(t)$ in terms of $M_Y(t)$ and $M_X(t)$.	Moment Generating Functions Describe the distribution of *X* if $M_X(t)=p_1e^{at}+p_2e^{bt}+p_3e^{ct}$.
Actex – Ch. 05	Actex – Ch. 05	Actex – Ch. 05
Percentiles Define the 100*p*-th percentile of a continuous random variable, *X*.	Skewness Give formula for skewness.	Chebyshev's Inequality State Chebyshev's Inequality.

If $X \geq 0$, then $E[X] = \int_0^\infty S(x)\,dx$	$E[X] = \sum h(x)\,f(x)$ $E[X] = \int_{-\infty}^{\infty} h(x)\,f(x)\,dx$	$E[X] = \sum x\,f(x)$ $E[X] = \int_{-\infty}^{\infty} x\,f(x)\,dx$
$E[aX + b] = a\,E[X] + b$ $\text{Var}[aX + b] = a^2\,\text{Var}[X]$	$\text{Var}[X] = E\left[(X - \mu)^2\right]$ $\text{Var}[X] = E[X^2] - (E[X])^2$	$E[X^n]$ $E\left[(X - \mu)^n\right]$
$\frac{d^2}{dt^2} \ln\left[M_X(t)\right]\Big\rvert_{t=0} = \text{Var}[X]$	$M_X(0) = 1$ $M'_X(0) = E[X]$ $M_X^{(n)}(0) = E[X^n]$	$M_X(t) = E\left[e^{tx}\right]$
X is discrete with three possible values: a, b, and c. Furthermore: $P[X = a] = p_1$ $P[X = b] = p_2$ $P[X = c] = p_3$	$M_S(t) = M_X(t) \cdot M_Y(t)$	$M_Y(t) = e^{bt} M_X(at)$
$P\left[\lvert X - \mu_X \rvert > r\,\sigma_X\right] \leq \frac{1}{r^2}$	$\frac{E\left[(X - \mu)^3\right]}{\sigma^3}$	The 100p-th percentile is the smallest number c_p for which $F(c_p) \geq p$

Actex – Ch. 05	Actex – Ch. 05	
Truncated Distributions $E[X \mid a<X<b]=?$ $E[X \mid X>k]=?$	Coefficient of Variation Define c_v .	
Actex – Ch. 06	Actex – Ch. 06	Actex – Ch. 06
Discrete Uniform Distribution Give $f(x)$. (Assuming $x=1,2,\ldots,N$.)	Discrete Uniform Distribution Give $E[x]$ and $\mathrm{Var}[x]$. (Assuming $x=1,2,\ldots,N$.)	Discrete Uniform Distribution Give $M_X(t)$. (Assuming $x=1,2,\ldots,N$.)
Actex – Ch. 06	Actex – Ch. 06	Actex – Ch. 06
Binomial Distribution Describe what X measures. Give $f(x)$.	Binomial Distribution Give $E[x]$ and $\mathrm{Var}[x]$.	Binomial Distribution Give $M_X(t)$.
Actex – Ch. 06	Actex – Ch. 06	Actex – Ch. 06
Poisson Distribution Describe what X measures. Give $f(x)$.	Poisson Distribution Give $E[x]$ and $\mathrm{Var}[x]$.	Poisson Distribution Give $M_X(t)$.

	$c_v = \frac{\sigma}{\mu}$	$E[X \mid a < X < b] = \frac{1}{F(b) - F(a)} \int_a^b x f(x)\, dx$ $E[X \mid X > k] = \frac{1}{S(k)} \int_k^\infty x f(x)\, dx$
$M_X(t) = \frac{e^t(e^{Nt} - 1)}{N(e^t - 1)}$ Note: This is the sum of a finite geometric series	$\mu = \frac{N+1}{2}$ $\sigma^2 = \frac{N^2 - 1}{12}$	$f(x) = \frac{1}{N}$ $x = 1, 2, \ldots, N$
$M_X(t) = (q + p e^t)^n$	$\mu = n p$ $\sigma^2 = n p q$	X = number of successes in n trials. $f(x) = \binom{n}{x} p^x q^{n-x}$ $x = 0, 1, \ldots, n$
$M_X(t) = e^{\lambda(e^t - 1)}$	$\mu = \lambda$ $\sigma^2 = \lambda$	X = number of times an event occurs in a unit of time $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

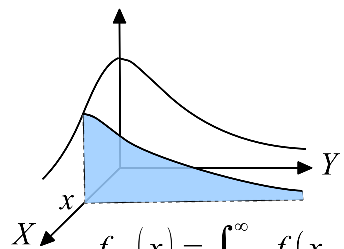
Actex – Ch. 06	Actex – Ch. 06	Actex – Ch. 06
Geometric Distribution Describe what X measures. Give $f(x)$.	Geometric Distribution Give $E[x]$ and $\text{Var}[x]$.	Geometric Distribution Give $M_X(t)$.
Actex – Ch. 06	**Actex – Ch. 06**	**Actex – Ch. 06**
Negative Binomial Distribution Describe what X measures. Give $f(x)$.	Negative Binomial Distribution Give $E[x]$ and $\text{Var}[x]$.	Negative Binomial Distribution Give $M_X(t)$.
Actex – Ch. 06	**Actex – Ch. 06**	**Actex – Ch. 06**
Hypergeometric Distribution Describe what X measures. Give $f(x)$.	Hypergeometric Distribution Give $E[x]$ and $\text{Var}[x]$.	Discrete Distributions What is the relationship between the Negative Binomial and Geometric distributions?
Actex – Ch. 06	**Actex – Ch. 06**	**Actex – Ch. 06**
Multinomial Distribution Give $f(x_1, x_2, \ldots, x_n)$.	Multinomial Distribution Give $E[x]$ and $\text{Var}[x]$.	Geometric Distribution State the consequences of the Memoryless Property.
Actex – Ch. 07	**Actex – Ch. 07**	**Actex – Ch. 07**
Continuous Uniform Distribution Give $f(x)$.	Continuous Uniform Distribution Give $E[x]$ and $\text{Var}[x]$.	Continuous Uniform Distribution Give $M_X(t)$.

$M_X(t) = \dfrac{p}{1-q e^t}$ Note: This is the sum of an infinite geometric series	$\mu = \dfrac{q}{p}$ $\sigma^2 = \dfrac{q}{p^2}$	X = number of failures prior to first success $f(x) = q^x p$ $x = 0,1,2,\ldots$
$M_X(t) = \left[\dfrac{p}{1-q e^t}\right]^r$	$\mu = \dfrac{r q}{p}$ $\sigma^2 = \dfrac{r q}{p^2}$	X = number of failures prior to success number r $f(x) = \binom{r+x-1}{x} p^r q^x$ $x = 0,1,2,\ldots$
$\text{NegBin}(r=1, p) \sim \text{Geom}(p)$	$\mu = \dfrac{n D}{T}$ $\sigma^2 = \dfrac{n D (T-D)(T-n)}{T^2 (T-1)}$	D desired objects; T objects total X = number of desired objects in a sample of size n (w/o rep.) $f(x) = \dfrac{\binom{D}{x}\binom{T-D}{n-x}}{\binom{T}{n}}$
$P[X=x+k \mid X \geq k] = P[X=x]$ $E[X \mid X \geq k] = E[X] + k$ $\text{Var}[X \mid X \geq k] = \text{Var}[X]$	$E[X_i] = n p_i$ $\text{Var}[X_i] = n p_i (1-p_i)$	$f(x_1, x_2, \ldots, x_n) =$ $\dfrac{n!}{x_1! \cdot x_2! \cdot \ldots \cdot x_k!} \cdot p_1^{x_1} \cdot p_2^{x_2} \cdot \ldots \cdot p_k^{x_k}$ $x_1 + x_2 + \ldots + x_k = n$
$M_X(t) = \dfrac{e^{bt} - e^{at}}{(b-a) \cdot t}$ Note: This is an easy integral.	$\mu = \dfrac{b+a}{2}$ $\sigma^2 = \dfrac{(b-a)^2}{12}$	$f(x) = \dfrac{1}{b-a}$ $a < x < b$

Actex – Ch. 07	Actex – Ch. 07	Actex – Ch. 07
Normal Distribution Give $f(x)$.	Normal Distribution Give $E[x]$ and $\text{Var}[x]$.	Normal Distribution Give $M_X(t)$.
Actex – Ch. 07	Actex – Ch. 07	Actex – Ch. 07
Exponential Distribution Give $f(x)$, $F(x)$, and $S(x)$.	Exponential Distribution Give $E[x]$ and $\text{Var}[x]$.	Exponential Distribution Give $M_X(t)$.
Actex – Ch. 07	Actex – Ch. 07	Actex – Ch. 07
Gamma Distribution Give $f(x)$.	Gamma Distribution Give $E[x]$ and $\text{Var}[x]$.	Gamma Distribution Give $M_X(t)$.
Actex – Ch. 07	Actex – Ch. 07	Actex – Ch. 07
Exponential Distribution State the consequences of the Memoryless Property.	Continuous Distributions State the relationship between the Gamma and Exponential distributions.	Distributions State the relationship between the Poisson and Exponential distributions.
Actex – Ch. 07	Actex – Ch. 07	
Integer Correction State the integer correction rule used when approximating a discrete distribution. with a normal dist.	Exponential Distribution Assume $Y_i \sim \text{Exp}(\lambda_i)$ and $Y = \min\{Y_1, \ldots, Y_n\}$. Describe distribution of *Y*.	Gamma Distribution Let $X \sim GAM(\alpha, \beta)$. Find $F_X(k)$ in terms of a a related Poisson distribution.

$M_X(t) = \exp\left[\mu t + \frac{\sigma^2 t^2}{2}\right]$	μ σ^2	$f(x) = \frac{1}{\sigma\cdot\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$ $-\infty < x < \infty$
$M_X(t) = \frac{\lambda}{\lambda - t}$	$\mu = \frac{1}{\lambda}$ $\sigma^2 = \frac{1}{\lambda^2}$	$f(x) = \lambda e^{-\lambda x}$ $F(x) = 1 - e^{-\lambda x}$ $S(x) = e^{-\lambda x}$ $x > 0$
$M_X(t) = \left(\frac{\beta}{\beta - t}\right)^\alpha$	$\mu = \frac{\alpha}{\beta}$ $\sigma^2 = \frac{\alpha}{\beta^2}$	$f(x) = \frac{\beta^\alpha \cdot x^{\alpha-1} \cdot e^{-\beta x}}{\Gamma(\alpha)}$ $x > 0$ Note: $\Gamma(\alpha) = (\alpha-1)!$ if $\alpha \in \mathbb{Z}$
X = time between events N = # of events in one unit of time If $X \sim \text{Exp}(\lambda)$, then $N \sim \text{Poisson}(\lambda)$ Also note: $\mu_X = \frac{1}{\lambda}$, and $\mu_N = \lambda$	$\text{Gamma}(\alpha=1, \beta=\lambda) \sim \text{Exp}(\lambda)$	$P[X > x+k \mid X > k] = P[X > x]$ $E[X \mid X \geq k] = E[X] + k$ $\text{Var}[X \mid X \geq k] = \text{Var}[X]$
Let $X \sim GAM(\alpha, \beta)$. Let $Y \sim POI(\lambda = \beta k)$. $F_X(k) = 1 - F_Y(\alpha - 1)$	$Y \sim \text{Exp}(\lambda_1 + \lambda_2 + \ldots + \lambda_n)$	If X is discrete and Y is normal, then: $P[a \leq X \leq b]$ $\approx P[a - 0.5 \leq Y \leq b + 0.5]$

Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Joint Distributions Define $F(x,y)$. (Discrete and Continuous)	Joint Distributions X, Y - jointly dist with pdf $f(x,y)$ Give $P[(x,y) \in R]$	Joint Distributions Given $F(x,y)$, find $f(x,y)$. (Continuous case)
Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Joint Distributions Given $f(x,y)$, find $E[h(x,y)]$. (Discrete and Continuous)	Marginal Distributions Given $f(x,y)$, find $f_X(x)$. (Continuous case)	Marginal Distributions Given $f(x,y)$, find $f_X(x)$ and $F_X(x)$. (Continuous case)
Actex – Ch. 08	Actex – Ch. 08	
Marginal Distributions Given $f(x,y)$, find $f_X(x)$ and $F_X(x)$. (Discrete case)	Marginal Expectation Given jointly distributed variables X and Y, find $E[X]$ (Discrete and Continuous)	
Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Conditional Distributions Given $f(x,y)$, find $g(x \mid y)$ and $h(y \mid x)$.	Conditional Distributions Given $f(x,y)$, find $P[a \le X \le b \mid Y = k]$.	Conditional Distributions Write $f(x,y)$ in terms of marginal and conditional pdf's.
Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Conditional Expectation Find $E[X \mid Y=k]$. (Discrete and Continuous)	Conditional Expectation Find $E[E[X \mid Y]]$.	Conditional Variance Find $\text{Var}[X \mid Y = k]$

$\frac{\partial^2}{\partial x \partial y} F(x, y) = f(x, y)$	$P[(x, y) \in R] = \iint_R f(s, t)\, dA$	$F(x, y) = \sum_{s \le x} \sum_{t \le y} f(s, t)$ $F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(s, t)\, dt\, ds$
$f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$ $F_X(x) = P[X \le x] = \int_{-\infty}^{x} f_X(t)\, dt$	 $f_X(x) = \int_{-\infty}^{\infty} f(x, y)\, dy$	$\sum_x \sum_y h(x, y) \cdot f(x, y)$ $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y) f(x, y)\, dx\, dy$
	$E[X] = \sum_x x f_X(x)$ $E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$	$f_X(x) = P[X = x] = \sum_y f(x, y)$ $F_X(x) = P[X \le x] = \sum_{t \le x} f_X(t)$
$f(x, y) = g(x \mid y) f_Y(y)$ $= h(y \mid x) f_X(x)$	$P[a \le X \le b \mid Y = k]$ $= \int_a^b g(x \mid k)\, dx$ $= \frac{1}{f_Y(k)} \int_a^b f(x, k)\, dx$	$g(x \mid y) = \frac{f(x, y)}{f_Y(y)}$ $h(y \mid x) = \frac{f(x, y)}{f_X(x)}$
$\text{Var}[X \mid Y = k] =$ $E[X^2 \mid Y = k] + (E[X \mid Y = k])^2$	$E_Y[E_X[X \mid Y]] = E[X]$	$E[X \mid Y = k] = \sum_x x\, g(x \mid k)$ $E[X \mid Y = k] = \int_{-\infty}^{\infty} x\, g(x \mid k)\, dx$

Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Law of Total Variance Represent $\text{Var}[X]$ using the Law of Total Variance.	Covariance 1) Give definition of $\text{Cov}[X, Y]$. 2) Explain the relationship between variance and covariance.	Correlation Coefficient Define $\rho(X, Y)$.
Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Covariance $\text{Cov}[aX, bY] = ?$ $\text{Cov}[X+a, Y+b] = ?$	Joint Distributions 1) Define $M_{X,Y}(t_1, t_2)$. 2) Give $E[X^n Y^m]$ in terms of $M_{X,Y}(t_1, t_2)$.	Bivariate Normal If X and Y have a bivariate normal distribution, then X, Y, $X \mid Y=k$, and $Y \mid X=k$ are all normal. $E[X \mid Y=k] = ?$ $\text{Var}[X \mid Y=k] = ?$
Actex – Ch. 08	Actex – Ch. 08	Actex – Ch. 08
Joint Distributions X and Y are indep if and only if: 1. $f(x, y) = ?$ 2. $F(x, y) = ?$ 3. $g(x \mid y) = ?$ and $h(y \mid x) = ?$	Joint Distributions If X and Y are independent, then: 1. $E[XY] = ?$ 2. $E[X \mid Y=k] = ?$ 3. $E[Y \mid X=k] = ?$	Joint Distributions If X and Y are independent, then: 1. $\text{Cov}[X, Y] = ?$ 2. $\rho_{X,Y} = ?$

$\rho(X,Y) = \frac{\text{Cov}[X,Y]}{\sigma_X \sigma_Y}$	$\text{Cov}[X,Y] = E[XY] - E[X]E[Y]$ $\text{Cov}[X,X] = \text{Var}[X]$	$\text{Var}[X] =$ $E_Y[\text{Var}[X \mid Y]] + \text{Var}_Y[E[X \mid Y]]$ See Actex – Section 8, #45 for an example.
$E[X \mid Y=k] =$ $\mu_X + \rho_{XY}\frac{\sigma_X}{\sigma_Y}(y - \mu_Y)$ $\text{Var}[X \mid Y=k] = \sigma_X^2(1 - \rho_{XY}^2)$	$M_{X,Y}(t_1,t_2) = E[e^{t_1 X + t_2 Y}]$ $E[X^n Y^m] =$ $\frac{\partial^{n+m}}{\partial^n t_1 \partial^m t_2} M_{X,Y}(t_1,t_2)\Big\vert_{t_1=t_2=0}$	$\text{Cov}[aX,\ bY] = ab\,\text{Cov}[X,\ Y]$ $\text{Cov}[X+a,\ Y+b] = \text{Cov}[X,\ Y]$
If X and Y are independent, then: 1. $\text{Cov}[X,Y] = 0$ 2. $\rho_{X,Y} = 0$ Note: The converse is not always true.	If X and Y are independent, then: 1. $E[XY] = E[X]E[Y]$ 2. $E[X \mid Y=k] = E[X]$ 3. $E[Y \mid X=k] = E[Y]$ Note: The converse is not always true.	X and Y are indep if and only if: 1. $f(x,y) = f_X(x) \cdot f_y(y)$, and prob. space is a rectangle 2. $F(x,y) = F_X(x) \cdot F_y(y)$ 3. $g(x \mid y) = f_X(x)$ and $h(y \mid x) = f_Y(y)$

Actex – Ch. 09	Actex – Ch. 09	Actex – Ch. 09
Single Variable Transformations $Y=u(X)$ and $X=v(Y)$. Find $f_Y(y)$ and $F_Y(y)$.	Multivariate Transformations Given a transformation $(X,Y)\rightarrow(U,V)$, find $g(u,v)$.	Sums of Random Variables Let $Y=\sum_{i=1}^{n} X_i$. Find $E[Y]$ and $\mathrm{Var}[Y]$.
Actex – Ch. 09	Actex – Ch. 09	Actex – Ch. 09
Sums of Random Variables Assume $X=\sum_{i=1}^{n} X_i$ and $Y=\sum_{i=1}^{m} Y_i$. Find $\mathrm{Cov}[X,Y]$.	Sums of Random Variables (Discrete Convolution Method) If $Y=X_1+X_2$ and $X_1,X_2\geq 0$, then $f_Y(y)=?$	Sums of Random Variables (Continuous Convolution Method) If $Y=X_1+X_2$, then $f_Y(y)=?$
Actex – Ch. 09	Actex – Ch. 09	Actex – Ch. 09
Central Limit Theorem $X_1,\ldots,X_n$ are IID and $Y=\sum_{i=1}^{n} X_i$. What does CLT say about distribution of Y?	Central Limit Theorem $X_1,\ldots,X_n$ are IID and $\bar{X}=\frac{1}{n}\sum_{i=1}^{n} X_i$ What does CLT say about distribution of $\bar{X}$?	Mixtures of Distributions Assume $X=\begin{cases} X_1 & \text{with prob } p_1 \\ X_2 & \text{with prob } p_2 \end{cases}$ Find $f(x)$, $E[X]$, and $E[X^2]$.
Actex – Ch. 09	Actex – Ch. 09	Actex – Ch. 09
Minimum and Maximum $X_1,\ldots,X_n$ are mutually independent. Find $F_{max}(x)$ and $S_{min}(x)$.	Order Statistics Let $Y_1,\ldots,Y_n$ be order statistics for independent observations of X. Find the joint pdf, $g(y_1,\ldots,y_n)$.	Order Statistics Let $Y_1,\ldots,Y_n$ be order statistics for independent observations of X. Find the marginal pdf, $g_k(t)$.
Actex – Ch. 10	Actex – Ch. 10	Actex – Ch. 10
Deductible Let $Y=\begin{cases} 0 & \text{if } X\leq d \\ x-d & if\ X>d \end{cases}$ Find $E[Y]$.	Policy Limit Let $Y=\begin{cases} X & \text{if } X\leq u \\ u & if\ X>u \end{cases}$ Find $E[Y]$.	Deductible and Policy Limit Let $Y=\begin{cases} 0 & \text{if } X\leq d \\ X-d & \text{if } d<X<d+u \\ u & \text{if } X>d+u \end{cases}$ Find $E[Y]$.

$E[Y] = E[X_1] + E[X_2] + \ldots + E[X_n]$ $\text{Var}[Y] = \sum \text{Var}[X_i] + 2\sum\sum \text{Cov}[X_i, X_j]$	$g(u,v) = f(x(u,v),\ y(u,v))\|J\|$ where $J = \begin{vmatrix} x_u & x_v \\ y_u & y_v \end{vmatrix}$	$f_Y(y) = f_X(v(y)) \cdot \|v'(y)\|$ $F_Y(y) = F_X(v(y))$
$f_Y(y) = \int_{-\infty}^{\infty} f(x_1,\ y - x_1) d_{x_1}$ $(\boldsymbol{X \perp Y}) \Rightarrow f_Y(y) = \int_{-\infty}^{\infty} f_1(x_1) f_2(y - x_1) d_{x_1}$ $(\boldsymbol{X_1, X_2 \geq 0}) \Rightarrow f_Y(y) = \int_0^y f(x_1, y - x_1) d_{x_1}$	$f_Y(y) = \sum_{x_1=0}^{y} f(x_1,\ y - x_1)$ If $X \perp Y$, then $f_Y(y) = \sum_{x_1=0}^{y} f_1(x_1) f_2(y - x_1)$	$\text{Cov}[X, Y] = \sum_n \sum_m \text{Cov}[X_i,\ Y_j]$
$f(x) = p_1 f_1(x) + p_2 f_2(x)$ $E[X] = p_1 E[X_1] + p_2 E[X_2]$ $E[X^2] = p_1 E[X_1^2] + p_2 E[X_2^2]$ Note: $\text{Var}[X] \neq p_1 \text{Var}[X_1] + p_2 \text{Var}[X_2]$	$E[\bar{X}] = \mu$ $\text{Var}[\bar{X}] = \sigma^2 / n$ If n is large, then $\bar{X}$ is approximately normal.	$E[Y] = n\mu$ $\text{Var}[Y] = n\sigma^2$ If n is large, then Y is approximately normal.
$g_k(t) = k\binom{n}{k}[F(t)]^{k-1}[S(t)]^{n-k} f(t)$	$g(y_1, \ldots, y_n) = n!\, f(y_1) \cdot f(y_2) \cdot \ldots \cdot f(y_n)$ where $y_1 \leq y_2 \leq \ldots \leq y_n$	$F_{max}(x) = F_1(x) \cdot F_2(x) \cdot \ldots \cdot F_n(x)$ $S_{min}(x) = S_1(x) \cdot S_2(x) \cdot \ldots \cdot S_n(x)$
$E[Y] = \int_d^{d+u} (x - d) f_X(x) dx + u S_X(d+u)$ $E[Y] = \int_d^{d+u} S_X(x) dx$	$E[Y] = \int_0^u x f_X(x) dx + u S_X(u)$ $E[Y] = \int_0^u S_X(x) dx$	$E[Y] = \int_d^{\infty} (x - d) f_X(x) dx$ $E[Y] = \int_d^{\infty} S_X(x) dx$